The Dynamics of Strategic Alliances: Theory and Experimental Evidence
Appendices

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A Examples of Models of Competition

Example 1: Cournot-quantity competition with linear demands and constant marginal costs. Each of the two firms produce and sell one good at a constant marginal cost of production $\tilde{c}$. Individual demand for each product, x_i, is given by

$$p_i = a - x_i - d\; x_j, \tag{1}$$

where x_j is the quantity produced by the partner firm j, and p_i denotes price. Parameter a is positive and $d \in [0, 1]$ captures the degree of substitution between the products. If $d = 0$ both goods are completely independent and each firm is a monopolist in an isolated market, while if $d = 1$ the goods are homogeneous. Firm i chooses x_i to maximize its profits given by

$$\pi_i = (a - x_i - dx_j - \tilde{c})\;\; x_i.$$

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Equilibrium (Cournot) quantities produced and gross profits are symmetric, and given by

$$x\left(\tilde{c},d\right)=\frac{a-\tilde{c}}{2+d}\text{ and }\pi\left(\tilde{c},d\right)=\left(\frac{a-\tilde{c}}{2+d}\right)^{2}. \tag{2}$$

Straightforward algebra shows that $\partial\pi\left(\tilde{c},d\right)/\partial\tilde{c}<0$, $\partial\pi\left(\tilde{c},d\right)/\partial d<0$ and $\partial\pi\left(\tilde{c},d\right)/\partial d\partial\tilde{c}>0$. Clearly, decreasing the marginal costs $\tilde{c}$ is exactly the same as increasing the market size a.

Example 2: Bertrand-price competition with linear demands and constant marginal costs. Again, each of the two firms produce and sell one good at a constant marginal cost of production $\tilde{c}$. From the indirect demand above (**??**), we can compute the direct demand functions,

$$x_i=\alpha-\beta p_i+\gamma p_{-i}, \tag{3}$$

where $\alpha=\frac{a}{1+d}$, $\beta=\frac{1}{(1+d)(1-d)}$, and $\gamma=\frac{d}{(1+d)(1-d)}$. Firm i chooses p_i to maximize its profits given by

$$\pi_i=\left(\alpha-\beta p_i+\gamma p_{-i}\right)\ (p_i-\tilde{c}).$$

Equilibrium (Bertrand) prices and gross profits are again symmetric, and given by

$$p\left(\tilde{c},d\right)=\frac{\alpha+\beta\tilde{c}}{2\beta-\gamma}\text{ and }\pi\left(\tilde{c},d\right)=2\beta\left(\frac{\alpha-\tilde{c}(\beta-\gamma)}{2\beta-\gamma}\right)^{2}. \tag{4}$$

Straightforward algebra shows that $\partial\pi\left(\tilde{c},d\right)/\partial\tilde{c}<0$, $\partial\pi\left(\tilde{c},d\right)/\partial d<0$ and $\partial\pi\left(\tilde{c},d\right)/\partial d\partial\tilde{c}>0$.

B Proofs

B.1 Proof of Propositions 1 and 2

Consider the row firm in Table 1. Notice that the assumption $\pi\left(c-kr,d\right)-\pi\left(c-r,d\right)>\pi\left(c-r,d\right)-\pi\left(c,d\right)$ implies that $k>1$, as the right-hand side is positive. In addition, we have that if the profits in (C,nC) are larger than those in (nC,nC) then we have that the profits in (C,C) are larger than those in (nC,C).

Notice first that, as shown by the table, the profits in (C,nC), gross of the participation costs, are larger than those of (nC,nC) if and only if $\pi\left(c-r,d\right)-\pi\left(c,d\right)\geq e$. Both terms in the left hand-side decrease as d increases, because $\partial\pi\left(\tilde{c},d\right)/\partial d<0$. But the derivative of the subtraction is negative because the second term decreases by less, given that $c-r<c$ and $\partial\pi(\tilde{c},d)/\partial d\partial\tilde{c}>0$. Therefore, there exists $d^*\in[0,1]$ such that the profits in (C,nC) are larger than those in (nC,nC) if and only if $d\leq d^*$.

Second, the profits in (C,C), gross of the participation costs, are larger than those of (nC,C) if and only if $\pi(c-kr,d)-\pi(c-r,d)\geq e$. Given that $\partial\pi(\tilde{c},d)/\partial\tilde{c}<0$, for each d there exists $k^*(d)$ such that the profits in (C,C) are larger than those of (nC,C) if and only if $k\geq k^*(d)$. By the same argument as above, the left-hand side of the inequality is decreasing in d. Therefore, $k^*(d)$ is increasing in d.

Third, the profits in (C,C), gross of the participation costs, are larger than those of (nC,nC) if and only if $\pi(c-kr,d)-\pi(c,d)\geq e$. Following the same arguments as above, for each d there exists $k^{**}(d)$, increasing in d, such that the profits in (C,C) are larger than those of (nC,nC) if and only if $k\geq k^{**}(d)$. Also, given that $\pi(c-r,d)>\pi(c,d)$, we have that $k^*(d)>k^{**}(d)$ for any d.

We now consider the various cases. First, suppose that $d<d^*$ and therefore the profits in (C,nC) are larger than those in (nC,nC). By the assumption, the profits in (C,C) are larger than those in (nC,C). Therefore, choosing C is a dominant strategy. Second, suppose now that $d>d^*$ and $k>k^*(d)$. By the previous arguments, the profits in (nC,nC) are larger than those in (C,nC) and the profits in (C,C) are larger than those of (nC,C). We have two equilibria, and given that $k>k^*(d)>k^{**}(d)$, the equilibria are Pareto-ranked. Third, suppose that $d>d^*$ and $k^*(d)>k$. We now have that the profits in (nC,nC) are larger than those in (C,nC) and the profits in (nC,C) are larger than those of (C,C). Therefore, choosing nC is a dominant strategy and (nC,nC) the resulting equilibrium. We have that if $k^*(d)>k>k^{**}(d)$ the equilibrium is Pareto-dominated but if $k^*(d)>k^{**}(d)>k$ it is not.

B.2 Proof of Proposition 3

With the assumed discount factor (0.9), the (inter-temporal) utility function for Firm i $(i=1,2)$ is given by

$$\sum_{\tau=1}^{\infty}(0.9)^{\tau-1}u_i(a_{1\tau},a_{2\tau}),$$

where $u_i(a_{1\tau},a_{2\tau})$ is Firm i's payoff at time τ when Firm 1 plays $a_{1\tau}$ and Firm 2 plays $a_{2\tau}$ in that time period. We show below that play of (C,C) forever can be sustained as a subgame perfect Nash equilibrium if both Firms follows a strategy specifying play of C at the outset, a play of E at every history except those where C was played in all previous periods, and a play of C at every history where C was played in all previous periods.

Take the most profitable deviation for Firm i. This would require it to play nC at some point t, with the corresponding response of the other firm. This deviation would yield the payoff on the left-hand side below. We want to show that it is less than the payoff from not deviating and, hence, achieving a play of (C, C) forever, which is the payoff on the right-hand side below:

$$\sum_{\tau=1}^{t-1}(0.9)^{\tau-1}5 + (0.9)^{t-1}6 + \sum_{\tau=t+1}^{\infty}(0.9)^{\tau-1}3 \leq \sum_{\tau=1}^{t-1}(0.9)^{\tau-1}5 + (0.9)^{t-1}5 + \sum_{\tau=t+1}^{\infty}(0.9)^{\tau-1}5.$$

This inequality simplifies to the following expression,

$$(0.9)^{t-1} \leq 2\sum_{\tau=t+1}^{\infty}(0.9)^{\tau-1},$$

which is satisfied for any t, as it can be rewritten as

$$1 \leq 2\sum_{\tau=t+1}^{\infty}(0.9)^{\tau-t} = 2\sum_{\tau=1}^{\infty}(0.9)^{\tau} = 2\frac{0.9}{0.1} = 18.$$

C Experimental Instructions

Subjects in our experiment were given instructions on different screens of the game, embedded in the z-tree program where their choices were also made. The following paragraphs appeared on separate screens. Wherever relevant, we indicate transitions between screens:

WELCOME to the Alliance experiment

- You will be asked repeatedly whether you want to form an alliance with a given partner and, if the alliance is formed, whether you wish to cooperate or not with your partner.
- During the experiment you will be matched with several different partners. With the first two (2) partners you will interact only once. Each time, your partner will be randomly chosen among all subjects in the room.

- With your third partner you will establish a long-term relationship: he/she will remain your partner until a random variable determines the end of the relationship. If this happens early on in the experiment, you will be matched with a new (fourth) partner for another long-term relationship, and so on.
- Each long-term relationship will last for an unknown number of periods. At the end of every period the computer randomly decides whether the relationship will end or continue for one more period. The probability of ending is 10% and that of continuing on to the next period is 90%.

IMPORTANT: If in two (2) hours the experiment has not yet ended, we will agree to meet again and continue play some other day.
All payoffs for your and your partner's decisions are given in Francs. You accumulate payoff in Francs until the end of the experiment. You will then receive 1 Euro for every 25 Francs you made in the experiment. You will also receive a 5 Euro show-up fee in addition to your experimental payoff.
Instructions specific to the one-shot games:

Your partner for this game will only play with you ONCE.
You must choose one of the following options:

- Enter in an alliance with your partner and cooperate while in the alliance.
- Enter in an alliance with your partner and **not** cooperate while in the alliance.
- Not enter in an alliance with your partner.

Your payoffs depend on your choice and your partner's choice:

- If either you or your partner chooses NOT to enter the alliance, each one of you gets **3 Francs**.
- If you and your partner both choose to enter, the alliance is formed and payoffs will depend on your choice to cooperate or not.
- If in the alliance you choose to cooperate and your partner not, you receive **-1 Francs**, while your partner receives **6 Francs**. Vice-versa if your partner cooperates while you don't.

- If both you and your partner choose to cooperate, each one of you receives **5 Francs**. If both you and your partner choose not to cooperate, each one of you receives **0 Francs**.

On the following screen subjects saw the payoff table and played game one. After seeing their payoff from game one, subjects came to a next screen where they were reminded that they had a new partner and played a second one-shot game (game two). They next faced the instructions for the repeated game (game three):

You will now start a long-term relationship

- From now on you will interact repeatedly with the same partner until chance determines the end of your relationship.
- Your partner for this long-term relationship has been randomly chosen among all the subjects of this experiment.
- The length of this relationship is unknown. At the end of every period the computer randomly decides whether it will end or continue. The probability that it will decide to end is 10%, while with 90% probability it will decide to continue to the next period.
- If by chance this long-term relationship is relatively short, you will proceed to form a new long-term relationship with a different partner. On the other hand, if it is very long, you may be asked to return on a different day to continue your interaction.

Your payoffs depend on your choice and your partner's choice:

- If either you or your partner chooses NOT to enter the alliance, each one of you gets **3 Francs forever!**

NOTICE: If in one period you or your partner choose NOT to enter the alliance, you will not be given the opportunity to choose again in future periods. This choice means that your payoff in every remaining period of the relationship will be equal to 3 Francs.

- If you and your partner both choose to enter, the alliance is formed and payoffs will depend on your choice to cooperate or not.

- If in the alliance you choose to cooperate and your partner not, you receive **-1 Francs** while your partner receives **6 Francs** . Vice-versa if your partner cooperates while you don't.
- If both you and your partner choose to cooperate, each one of you receives **5 Francs**. If both you and your partner choose not to cooperate, each one of you receives **0 Francs**.

On the following screen subjects saw the payoff table for the long-term relationship plus warnings for its start:

Long-term Relationship

- You will now start a long-term relationship.
- From now on you will interact repeatedly with the **same** partner until chance determines the end of your relationship.

Subjects who chose or their partners chose not to enter the alliance in some past period, did not have an opportunity to choose again and were instead given the following screen: **Your partnership was ended in a previous period. Hence, you cannot make any choices this period.**